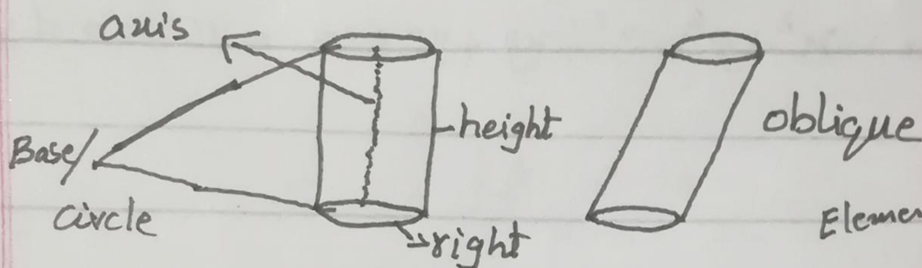
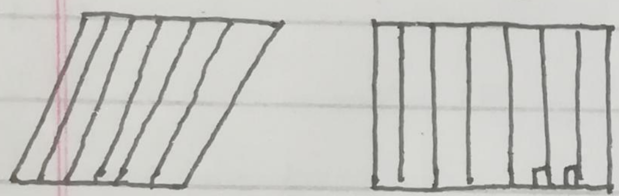
lecture 35

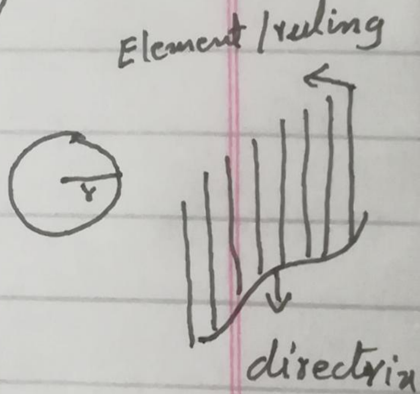
Cylinder

1 A cylinder is a closed solid that has two Parallel bases connected by a curved surface.

2 A surface consisting of all the points on all The lines which are parallel to a given line and which pass Through a fixed ~~line~~/ Plane curve in a plane not ~~Pat~~ parallel to the given line







1. Right cylinder

When the Two bases are exactly over each other and axis is a right angle to The base.

② Oblique cylinder

If one base is displaced sideways, the axis is not at right angle to The bases. bases are parallel but not directly over each other

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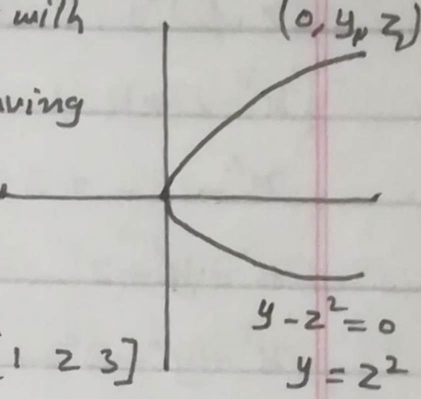
Find an equation of the cylinder with directrix $C: y-z^2=0$ and having elements parallel to $\bar{n}=[1\ 2\ 3]$.



$[x\ y\ z] = [0\ \ y_1\ \ z_1] + t[1\ 2\ 3]$

$[x\ y\ z] = [0\ \ y_1\ \ z_1] + [t\ \ 2t\ \ 3t]$

$[x\ y\ z] = [0+t\ \ \ y_1+2t\ \ \ z_1+3t]$

$x = t \quad , y = y_1+2t \quad , \quad z = z_1+3t$

$y = y_1+2x \qquad\qquad z = z_1+3x$

$y-2x = y_1 \qquad\qquad z-3x = z_1$

$y = y_1 \quad , \quad z = z_1$

$y-z^2=0 \quad , \quad y_1-z_1^2=0$

$y_1 = z_1^2$

$y-2x = (z-3x)^2$

If Trace of cylinder at $x=d$

$y-2d = (z-3d)^2$

$[(y-2d)-(z-3d)^2]=0$

Find an equation of the right cylinder whose directrix to the circle with centre $C(5,3,0)$

$r=4$

Circle equation

$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$

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$$(x-5)^2+(y-3)^2+(z-0)^2=4^2$$

$$[x^2-2(x)(5)+5^2]+[y^2-2(y)(3)+3^2]+z^2=16$$

$$x^2-10x+25+y^2-6y+9+z^2-16=0$$

$$x^2+y^2+z^2-10x-6y+25+9-16=0$$

$$x^2+y^2+z^2-10x-6y+34-16=0$$

$x^2+y^2+z^2-10x-6y+18=0$ is equation of directrix.

Elliptic cylinder: Directrix is ellips

An elliptic cylinder is a set of points (x,y,z) satisfying the equation

$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1 \quad xy\text{-plane}, z=0$$

$$\left(\frac{y}{a}\right)^2+\left(\frac{z}{b}\right)^2=1 \quad yz\text{- plane}, x=0$$

$$y=0$$

$$\frac{x^2}{a^2}+\frac{z^2}{b}=1 \quad , \left(\frac{x}{a}\right)^2+\left(\frac{z}{b}\right)^2=1 \quad xz\text{-plane}$$

Example

Discus the surface

$$8x^2+15y^2-5=0$$

Solution

$$8x^2+15y^2=5$$

$$\frac{8x^2}{5}+\frac{15y^2}{5}=\frac{5}{5}$$

$$\frac{8x^2}{5}+\frac{3y^2}{1}=1$$

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$$\frac{x^2}{5/8} + \frac{y^2}{1/3} = 1$$

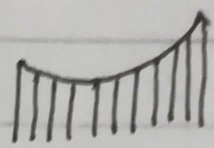
So, it is elliptic cylinder

Hyperpolic cylinder: Directrix is Hyperbola

An hyperbolic cylinder is set of points (x, y, z) satisfying the equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad xy\text{-plane}, \; z = 0$$

$$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$$



$$\frac{y^2}{a^2} + \frac{z^2}{b^2} = 1 \quad yz\text{-plane}, \; x = 0$$

$$\frac{x^2}{a^2} - \frac{z^2}{b^2} = 1 \quad xz\text{-plane}, \; y = 0$$

Example:

Discus the surface

$$5x^2 - 12y^2 - 7 = 0$$

Solution

$$5x^2 - 12y^2 = 7$$

$$\frac{5x^2}{7} - \frac{12y^2}{7} = \frac{7}{7}$$

$$\frac{x^2}{7/5} - \frac{y^2}{7/12} = 1$$

So, it is a Hyperbolic cylinder

Parabolic cylinder

Directrix is Parabola.

a parabolic cylinder is set of points (x,y,z) satisfying the equations

$x = cz^2$,	$x = cy^2$	xy-plane, $z = 0$
$y = cx^2$,	$y = cz^2$	yz-plane, $x = 0$
$z = cy^2$	$z = cx^2$	xz-plane, $y = 0$

lecture 37

Ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}+\frac{(z-l)^2}{c^2}=1 \quad \text{centre } (h,k,l)$$

$$\frac{(x-0)^2}{a^2}+\frac{(y-0)^2}{b^2}+\frac{(z-0)^2}{c^2}=1 \quad \text{centre } (0,0,0)$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$a=b=c$$

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{a^2}=1$$

$$x^2+y^2+z^2=a^2 \qquad \times\ a^2 \text{ on B.S}$$

centre $= C(0,0,0)$, $r=a$

Spheroid

If $a=b$, $a\neq c$, $b\neq c$ spheroid

If $a=b$, $c<a$, $c<b$ oblate

If $a=b$, $c>a$, $c>b$ Polate

Properties

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

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Symmetry

It is symmetric about x-axis, y-axis, z-axis, xy-plane, yz-plane, zx-plane and origin because x, y and z has even power in given equation.

(i) If $f(x,y,z)=f(x,-y,-z)$ — x-axis

(ii) If $f(x,y,z)=f(-x,y,-z)$ — y-axis

(iii) If $f(x,y,z) = f(-x,-y,z)$ — z-axis

iv If $f(x,y,z) = f(x,y,-z)$ — xy-plane

v If $f(x,y,z) = f(-x,y,z)$ — yz-plane

vi If $f(x,y,z) = f(x,-y,z)$ — xz-plane

vii If $f(x,y,z) = f(-x,-y,-z)$ — about origin

$$f(x,y,z)=0 \;,\; \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$f(x,-y,-z)=0 \;,\; \frac{x^2}{a^2}+\frac{(-y)^2}{b^2}+\frac{(-z)^2}{c^2}=1$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

Traces (i) xy-plane , $z=0$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{is ellips.}$$

(ii) yz-plane , $x=0$

$$\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \quad \text{is ellips.}$$

(iii) xz-plane , $y=0$

$$\frac{x^2}{a^2}+\frac{z^2}{c^2}=1 \quad \text{is ellips.}$$

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Intercept $\quad \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

I x-Intercept , $y=0$, $z=0$ $\quad (a,0,0)$

$\frac{x^2}{a^2}=1$, $x^2=a^2$, $x=\pm a$ $\quad (-a,0,0)$

II y-Intercept $x=0$, $z=0$ $\quad (0,b,0)$

$\frac{y^2}{b^2}=1$, $y^2=b^2$, $y=\pm b$ $\quad (0,-b,0)$

III z-Intercept $x=0$, $y=0$ $\quad (0,0,c)$

$\frac{z^2}{c^2}=1$, $z^2=c^2$, $z=\pm c$ $\quad (0,0,-c)$

Example Identify the surface

$100x^2+25y^2+4z^2-400x-150y-8z+529=0$

and find its traces and intercepts.

Solution

$$100x^2-400x+25y^2-150y+4z^2-8z=-529$$

$$\left[(10x)^2-2(10x)(20)+(20)^2\right]+\left[(5y)^2-2(5y)(15)+(15)^2\right]+\cdots$$

$$+\left[(2z)^2-2(2z)(2)+2^2\right]=-529+(20)^2+(15)^2+2^2$$

$$(10x-20)^2+(5y-15)^2+(2z-2)^2=-529+400+225+2^2$$

$$(10)^2(x-2)^2+(5)^2(y-3)^2+(2)^2(z-1)^2=100$$

$$100(x-2)^2+25(y-3)^2+4(z-1)^2=100$$

$$\frac{\cancel{100}(x-2)^2}{\cancel{100}}+\frac{\overset{1}{\cancel{25}}(y-3)^2}{\underset{4}{\cancel{100}}}+\frac{\overset{1}{\cancel{4}}(z-1)^2}{\underset{25}{\cancel{100}}}=\frac{100}{100}$$

$$\frac{(x-2)^2}{1}+\frac{(y-3)^2}{4}+\frac{(z-1)^2}{25}=1$$

$$\frac{(x-2)^2}{1^2}+\frac{(y-3)^2}{2^2}+\frac{(z-1)^2}{5^2}=1$$

is ellipsoid

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}+\frac{(z-l)^2}{c^2}=1$$

centre $C(h,k,l)=C(2,3,1)$

Traces xy-plane, $z=0$

$$\frac{(x-2)^2}{1^2}+\frac{(y-3)^2}{2^2}+\frac{(0-1)^2}{5^2}=1$$

$$\frac{(x-2)^2}{1^2}+\frac{(y-3)^2}{2^2}+\frac{1}{25}=1$$

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$$\frac{(x-2)^2}{1^2}+\frac{(y-3)^2}{2^2}=1-\frac{1}{25}$$

$$\frac{(x-2)^2}{1^2}+\frac{(y-3)^2}{2^2}=\frac{25-1}{25}=\frac{24}{25}$$

$$\frac{25}{24}\times\frac{(x-2)^2}{1}+\frac{25}{24}\times\frac{(y-3)^2}{4}=\frac{24}{25}\times\frac{25}{24}$$

$$\frac{25(x-2)^2}{24}+\frac{25(y-3)^2}{96}=1 \quad \text{is ellips}$$

<u>yz-Plane</u> $x=0$

$$\frac{(0-2)^2}{1^2}+\frac{(y-3)^2}{2^2}+\frac{(z-1)^2}{5^2}=1$$

$$\frac{4}{1}+\frac{(y-3)^2}{2^2}+\frac{(z-1)^2}{5^2}=1$$

$$\frac{(y-3)^2}{2^2}+\frac{(z-1)^2}{5^2}=1-4=-3 \quad \text{No trace}$$

<u>xz-Plane</u> $y=0$

$$\frac{(x-2)^2}{1^2}+\frac{(0-3)^2}{2^2}+\frac{(z-1)^2}{5^2}=1$$

$$\frac{(x-2)^2}{1^2}+\frac{9}{4}+\frac{(z-1)^2}{5^2}=1$$

$$\frac{(x-2)^2}{1^2}+\frac{(z-1)^2}{5^2}=1-\frac{9}{4}$$

$$\frac{(x-2)^2}{1^2}+\frac{(z-1)^2}{5^2}=\frac{4-9}{4}=\frac{-5}{4}$$

No trace

Intercept $\quad x$-Intercept $\quad y=0,\ z=0$

$$\frac{(x-2)^2}{1^2}+\frac{(0-3)^2}{2^2}+\frac{(0-1)^2}{5^2}=1$$

$$(x-2)^2+\frac{9}{4}+\frac{1}{25}=1$$

$$(x-2)^2=1-\frac{9}{4}-\frac{1}{25}$$

$$(x-2)^2=\frac{100-225-4}{100}$$

$$(x-2)^2=-\frac{129}{100}<0$$

No x-Intercept

y-Intercept $\quad x=0,\ z=0$

$$\frac{(0-2)^2}{1}+\frac{(y-3)^2}{4}+\frac{(0-1)^2}{25}=1$$

$$4+\frac{(y-3)^2}{4}+\frac{1}{25}=1$$

$$\frac{(y-3)^2}{4}=1-\frac{1}{25}-4$$

$$=\frac{25-1-100}{25}=\frac{-76}{25}<0$$

No-y-Intercept

z-Intercept $\quad x=0,\ y=0$

$$\frac{(0-2)^2}{1}+\frac{(0-3)^2}{4}+\frac{(z-1)^2}{25}=1$$

$$4+\frac{9}{4}+\frac{(z-1)^2}{25}=1$$

$$\frac{(z-1)^2}{25}=1-\frac{9}{4}-4=\frac{4-9-16}{4}$$

$$=\frac{-21}{4}<0$$

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No z-Intercept

Example Discuss the symmetry of ellipsoid

$$x^2 + \frac{y^2}{9} + \frac{(z-1)^2}{4} = 1$$

Solution

(1) xy - Plane $\quad z = -z$

$$\frac{x^2}{} + \frac{y^2}{9} + \frac{(-z-1)^2}{4} = 1$$

$$x^2 + \frac{y^2}{9} + \frac{(-1)^2(z+1)^2}{4} = 1$$

$$x^2 + y^2/9 + \frac{(z+1)}{4} = 1$$

$$f(x,y,z) \neq f(x,y,-z)$$

It is not symmetric about xy - Plane, x-axis y-axis and origin.

It is symetric about xz-plane, yz-plane and z-axis because x and y has even power.

yz - Plane $\quad x = -x$

$$(-x)^2 + \frac{y^2}{9} + \frac{(z-1)^2}{4} = 1$$

$$x^2 + \frac{y^2}{9} + \frac{(z-1)^2}{4} = 1$$

xz - plane , $\quad y = -y$

$$x^2 + \frac{(-y)^2}{9} + \frac{(z-1)^2}{4} = 1$$

$$x^2 + y^2/9 + \frac{(z-1)^2}{4} = 1$$

z - axis $\quad y = -y \; , \; x = -x$

$$(-x)^2 + (-y)^2/9 + \frac{(z-1)^2}{4} = 1$$

$$x^2 + y^2/9 + \frac{(z-1)^2}{4} = 1$$

Lecture 38

Elliptic paraboloid

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=c(z-l)$$

$$v(h,k,l)=v(0,0,0)$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=cz^2$$

if $c>0$ then paraboloid lies above xy-Plane

if $c<0$ Then paraboloid lies below xy-plane

$\frac{x^2}{4}+\frac{y^2}{9}=z$, $c=1$, open along z-axis, above xy-plane

$\frac{x^2}{4}+\frac{y^2}{9}=-z$, $c=-1$, open along z-axis, below xy-plane

$\frac{x^2}{a^2}+\frac{z^2}{b^2}=y \Rightarrow$ open along y-axis

$\frac{y^2}{a^2}+\frac{z^2}{b^2}=x \Rightarrow$ open along x-axis

1 Symmetry

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=z$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-z=0$$

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It is not symmetric about

$f(x,y,z)\neq f(x,y,-z)$ xy-plane

$f(x,y,z)\neq f(x,-y,-z)$ x-axis

$f(x,y,z)\neq f(-x,y,-z)$ y-axis

$f(x,y,z)\neq f(-x,-y,-z)$ about origin

because z has no even power in given equation.

It is symmetric about

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$f(x,y,z)=f(-x,y,z)$ $-yz$-Plane

$f(x,y,z)=f(x,-y,z)$ xz-Plane

$f(x,y,z) = f(-x,-y,z)$ z-axis

because x and y have even power in given equation

Traces $\frac{x^2}{a^2} + \frac{y^2}{b^2} = z$

(i) xy-Plane , $z = 0$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$ which is possible

only $x = 0$, $y = 0$ trace is origin in xy-plane

$z = 1$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is ellipse

(ii) yz-plane $x = 0$

$0 + \frac{y^2}{b^2} = z$, $y^2 = b^2 z$, $y = \pm b\sqrt{z}$

is parabola.

(iii) xz-plane $y = 0$

$\frac{x^2}{a^2} = z$, $x^2 = a^2 z$, $x = \pm a\sqrt{z}$

is parabola.

Intercept

x-Intercept $y = 0$, $z = 0$

$\frac{x^2}{a^2} + 0 = 0$ Then $x = 0$ $(0,0,0)$

y-Intercept $x = 0$, $z = 0$

$0 + \frac{y^2}{b^2} = 0$, Then $y = 0$ $(0,0,0)$

z-Intercept $x = 0$, $y = 0$

$0 + 0 = z$, Then $z = 0$ $(0,0,0)$

Boundedness

Elliptic paraboloid is not bounded

Example

Identify the surface

$25x^2+y^2-2y-25z+1=0$ and find its traces in the coordinate planes and all intercept.

Solution

$$25x^2+y^2-2y-25z+1=0$$

$$25x^2+y^2-2y+1=25z$$

$$25x^2+(y-1)^2=25z$$

$$\frac{25x^2}{25}+\frac{(y-1)^2}{25}=\frac{25z}{25}$$

$$x^2+\frac{(y-1)^2}{25}=z$$

$$\frac{x^2}{1^2}+\frac{(y-1)^2}{5^2}=z \quad \text{is elliptic paraboloid}$$

Traces

xy-Plane $z=0$

$$x^2+\frac{(y-1)^2}{5^2}=0 \quad \text{which is possible}$$

only $x=0$ and $y-1=0$, $y=1$

$$(0,1,0)$$

yz-Plane $x=0$

$$0+\frac{(y-1)^2}{25}=z, \quad (y-1)^2=25z$$

It is Parabola

xz-plane, $y=0$

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$$x^2+\frac{(0-1)^2}{25}=z, \quad x^2+\frac{1}{25}=z$$

$$x^2+\frac{1}{25}=z, \quad x^2=z-\frac{1}{25}$$

$25x^2+1=25z$ is parabola

Intercept

(i) <u>x - Intercept</u> $y=0,\ z=0$

$$x^2+\left(\frac{(0-1)^2}{25}\right)=0\ ,\ x^2+\frac{1}{25}=0\ ,\ x^2=-\frac{1}{25}$$

which is impossible means No - Intercept x-axis

(ii) <u>y - Intercept</u> $x=0,\ z=0$

$$0+\frac{(y-1)^2}{25}=0\ ,\ (y-1)^2=0\ ,\ y-1=0\ ,\ y=1$$

$$(0,1,0)$$

iii <u>z - Intercept</u> $x=0,\ y=0$

$$0+\frac{(0-1)^2}{25}=z\ ,\ \frac{1}{25}=z\ ,\ \left(0,0,\frac{1}{25}\right)$$

Hyperbolic paraboloid

$$\frac{y^2}{b^2}-\frac{x^2}{a^2}=z$$

Traces

<u>xy-plane</u> $z=0$

$$\frac{y^2}{b^2}=0+\frac{x^2}{a^2}\ ,\ \frac{y^2}{b^2}=\frac{x^2}{a^2}$$

$y^2=\frac{b^2}{a^2}x^2$, $y=\pm\frac{b}{a}x$ pair of intersecting lines.

<u>xz-plane</u> $y=0$

$-\frac{x^2}{a^2}=z$, $x^2=-a^2z$ parabola

<u>yz-plane</u> $x=0$

$\frac{y^2}{b^2}=z$, $y^2=b^2z$ Parabola

Symmetry

It is not symmetric about xy-plane, x-axis, y-axis and origin because z has not even power in equation

It is symmetric about yz-Plane, xz-Plane and z-axis because x and y have even power in given equation

<u>Intercept</u> $\quad \dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = z$

<u>x-Intercept</u> $\quad y=0,\ z=0$ then $x=0 \quad (0,0,0)$

<u>y-Intercept</u> $\quad x=0, z=0$ Then $y=0 \quad (0,0,0)$

<u>z-Intercept</u> $\quad x=0, y=0,$ Then $z=0 \quad (0,0,0)$

<u>Example</u> Identify the surface

$$-x^2+4y^2-4x-16y-4z+12=0$$

find Traces and Intercepts.

<u>Solution</u> $\quad -x^2+4y^2-4x-16y+12=4z$

$$-x^2-4x+4y^2-16y+12=4z$$

$$(4y^2-16y)+12-(x^2+4x)=4z$$

$$[(2y)^2-2(2y)(4)+(4)^2]-[x^2+2(x)(2)+2^2]=4z+4^2-2^2-12$$

$$(2y-4)^2-(x+2)^2=4z+16-4-12$$

$$2^2(y-2)^2-(x+2)^2=4z$$

$$\frac{4(y-2)^2}{4}-\frac{(x+2)^2}{4}=\frac{4z}{4}$$

$$(y-2)^2-\frac{(x+2)}{4}=z$$ It is a Hyperbolic paraboloid.

Traces $(y-2)^2 - \frac{(x+2)^2}{4} = z$

xy-Plane $z=0$

$$(y-2)^2 - \frac{(x+2)^2}{4} = 0 \quad , \quad (y-2)^2 = \frac{(x+2)^2}{4}$$

$$y-2 = \pm \frac{(x+2)}{2}$$

$y-2 = \frac{(x+2)}{2}$	$y-2 = -\frac{(x+2)}{2}$
$2(y-2) = x+2$	$2y-4 = -x-2$
$2y-4 = x+2$	$x+2y-4+2=0$
$0 = x-2y+2+4$	$x+2y-2=0$
$x-2y+6=0$	Pair of intersecting line

yz-Plane $x=0$

$$(y-2)^2 - \frac{(0+2)^2}{4} = z \quad , \quad (y-2)^2 = z + \frac{4}{4}$$

$(y-2)^2 = z+1$ is parabola.

xz-plane $y=0$

$$(0-2)^2 - \frac{(x+2)^2}{4} = z \quad , \quad (-2)^2 - z = \frac{(x+2)^2}{4}$$

$(x+2)^2 = 4(4-z)$, $(x+2)^2 = 16-4z$ is Parabola.

Intercepts $(y-2)^2 - \frac{(x+2)^2}{4} = z$

x- Intercepts $y=0$, $z=0$

$$(0-2)^2 - \frac{(x+2)^2}{4} = 0 \quad , \quad 4 = \frac{(x+2)^2}{4}$$

$(x+2)^2 = 16$, $x+2 = \pm 4$

$x+2 = 4$	$x+2 = -4$
$x = 4-2$	$x = -4-2$
$x = 2$	$x = -6$
$(2,0,0)$	$(-6,0,0)$

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<u>y-Intercept</u> $x=0,\ z=0$

$(y-2)^2-\frac{(0+2)^2}{4}=0 \quad ,(y-2)^2=\frac{4}{4}$

$(y-2)^2=1 \quad , \quad y-2=\pm 1 \quad , \quad y-2=1$

$y=2+1 \quad , \boxed{y=3} , \quad y-2=-1 \quad , \quad y=-1+2$

$(0,3,0) \qquad \boxed{y=1} \quad (0,1,0)$

<u>z-Intercept</u> $x=0,\ y=0$

$(0-2)^2-\frac{(0+2)^2}{4}=z, \quad 4-\frac{4}{4}=z$

$z=4-1=3 \quad , \quad (0,0,3)$

Lecture - 39

<u>Hyperboloid of one sheet</u>

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}-\frac{(z-l)^2}{c^2}=1 \quad$ centre $=C(h,k,l)$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \quad C(0,0,0)$, along z-axis

$\frac{x^2}{a^2}+\frac{z^2}{c^2}-\frac{y^2}{b^2}=1 \quad$ along y-axis

$\frac{y^2}{b^2}+\frac{z^2}{c^2}-\frac{x^2}{a^2}=1 \quad$ along x-axis

1 <u>Symmetry</u>

It is symmetric about x-axis, y-axis z-axis, xy-plane, yz-plane, xz-plane and origin because x, y and z have even power in given equation.

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \quad$ along z-axis

$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \quad$ along y-axis

$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \quad$ along x-axis

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y-Intercept $x=0,\ z=0$

$(y-2)^2 - \frac{(0+2)^2}{4} = 0$, $(y-2)^2 = \frac{4}{4}$

$(y-2)^2 = 1$, $y-2 = \pm 1$, $y-2 = 1$

$y = 2+1$, $\boxed{y=3}$, $y-2=-1$, $y=-1+2$

$(0,3,0)$ $\boxed{y=1}$ $(0,1,0)$

z-Intercept $x=0,\ y=0$

$(0-2)^2 - \left(\frac{0+2}{4}\right)^2 = z$, $4 - \frac{4}{4} = 2$

$z = 4-1 = 3$, $(0,0,3)$

Lecture - 39

Hyperboloid of one sheet is surface

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} - \frac{(z-l)^2}{c^2} = 1$ centre $= C(h,k,l)$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ $C(0,0,0)$, open along z-axis

$\frac{x^2}{a^2} + \frac{z^2}{c^2} - \frac{y^2}{b^2} = 1$ open along y-axis

$\frac{y^2}{b^2} + \frac{z^2}{c^2} - \frac{x^2}{a^2} = 1$ open along x-axis

1 **Symmetry**

It is symmetric about x-axis, y-axis z-axis, xy-plane, yz-plane, xz-plane and origin because x, y and z have even power in given equation.

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ open along z-axis

$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ open along y-axis

$-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ open along x-axis

(2) <u>Traces</u> $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

(i) <u>xy-plane</u> $z=0$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is ellips

(ii) <u>xz-plane</u> $y=0$

$\frac{x^2}{a^2}-\frac{z^2}{c^2}=1$ is Hyperbola

iii <u>yz-plane</u> $x=0$

$\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ is Hyperbola

(3) <u>Intercepts</u> $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

<u>x-Intercept</u> $y=0,\ z=0$

$\frac{x^2}{a^2}=1$, $x^2=a^2$, $x=\pm a$

$(a,0,0)$, $(-a,0,0)$

<u>y-Intercepts</u> $x=0,\ z=0$

$\frac{y^2}{b^2}=1$, $y^2=b^2$, $y=\pm b$, $(0,b,0),(0,-b,0)$

<u>z-Intercept</u> $x=0$, $y=0$

$-\frac{z^2}{c^2}=1$, $z^2=-c^2$, $z=\sqrt{-c^2}$

No, real set so, not z-Intercept

(4) <u>Boundedness</u>

Hyperboloid of one sheed is unbounded. It is not bounded

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<u>Example</u> Indetify the surface

$4x^2+y^2-4z^2-16x-4y+8z+12=0$

and find its centre and Intercepts.

<u>Solution</u> $4x^2-16x+y^2-4y-4z^2+8z=-12$

$$\left[4x^2-2(2x)(4)+4^2\right]+\left[y^2-2(y)(2)+2^2\right]-\left[4z^2-2(2z)(2)+2^2\right]=-12+4^2+2^2-2^2$$

$$(2x-4)^2+(y-2)^2-(2z-2)^2=-12+16+4-4$$

$$2^2(x-2)^2+(y-2)^2-2^2(z-1)^2=4$$

$$\frac{4(x-2)^2}{4}+\frac{(y-2)^2}{4}-\frac{4(z-1)^2}{4}=\frac{4}{4}$$

$$(x-2)^2+\frac{(y-2)^2}{4}-(z-1)^2=1$$

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}-\frac{(z-l)^2}{c^2}=1$$

centre $= C(h,k,l) = C(2,2,1)$ is centre of hyperboloid of one sheet

Intercepts $\quad 4x^2+y^2-4z^2-16x-4y+8z+12=0$

x-Intercept $\quad y=0,\ z=0$

$$4x^2-16x+12=0,\quad 4(x^2-4x+3)=0$$

$$x^2-4x+3=0,\quad x^2-3x-x+3=0$$

$$x(x-3)-1(x-3)=0,\quad (x-3)(x-1)=0$$

$x-3=0,\ x=3$ $\quad$ | $\quad x-1=0,\ x=1$

$(3,0,0)$ $\quad (1,0,0)$

y-Intercept $\quad x=0,\ z=0$

<u>y-Intercept</u> $x=0, z=0$

$y^2-4y+12=0$, $dic = b^2-4ac = (-4)^2-4(1)(12)$

$dic = 16-48 = -32$ No real set

So, No y-intercepts

<u>z-Intercept</u> $x=0, y=0$

$-4z^2+8z+12=0$, $-4(z^2-2z-3)=0$

$z^2-2z-3=0$, $z^2-3z+z-3=0$, $z(z-3)+1(z-3)=0$

$(z-3)(z+1)=0$, $z-3=0, z=3$ $(0,0,3)$

$z+1=0$, $z=-1$, $(0,0,-1)$

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Short Lecture 39
Hyperboloid of one & two sheets
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Hyperboloid of Two sheet

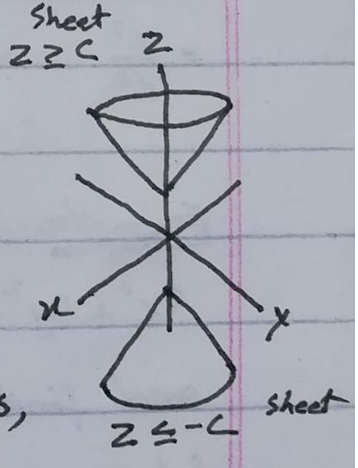
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}-\frac{(z-l)^2}{c^2}=-1$$, centre $= C(h,k,l)$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$ open along z-axis centre $=(0,0,0)$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$$ open along y-axis $(0,0,0)$

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$$ open along x-axis $(0,0,0)$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}-1$$



Possible if $\frac{z^2}{c^2}-1 \geq 0$ or, $\frac{z^2}{c^2} \geq 1$, $z^2 \geq c^2$

$z \leq -c$, $z \geq c$

<u>Symmetry</u> $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$

It is symmetric about x-axis, y-axis, z-axis, xy-plane, yz-plane, xz-plane and origin because x, y and z has even powers in given equation.

(2) **Traces** $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$

(i) xy-plane $z=0$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=-1 \quad \text{No trace } xy\text{-plane}$$

(ii) yz-plane $x=0$

$$\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1 \;,\; +\left(\frac{z^2}{c^2}-\frac{y^2}{b^2}\right)=+1$$

$$\frac{z^2}{c^2}-\frac{y^2}{b^2}=1 \quad \text{is hyperbola}$$

(iii) xz-plane $y=0$

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}=-1 \;,\; \frac{z^2}{c^2}-\frac{x^2}{a^2}=1 \text{ is hyperbola}$$

(3)

Intercepts $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$

(i) x-intercept $y=0,\ z=0$

$$\frac{x^2}{a^2}=-1 \;,\; x^2=-a^2 \quad \text{No real so Not intercept}$$

(ii) y-Intercept $x=0,\ z=0$

$$\frac{y^2}{b^2}=-1 \;,\; y^2=-b^2 \;,\; \text{No real , not intercepts}$$

(iii) z-intercept $x=0\ ,\ y=0$

$$+\frac{z^2}{c^2}=+1 \;,\; z^2=c^2 \;,\; z=\pm c$$

$$(0,0,c)\;,\;(0,0,-c)$$

Example Identify the surface

$$9x^2+9y^2-z^2-18x-18y+2z+26=0$$

and find its centre and traces

Solution

$$9x^2-18x+9y^2-18y-z^2+2z=-26$$

$$\left[(3x)^2-2(3x)(3)+3^2\right]+\left[(3y)^2-2(3y)(3)+3^2\right]-\left[z^2-2(z)(1)+1^2\right]$$

$$=-26+3^2+3^2-1^2$$

$$(3x-3)^2+(3y-3)^2-(z-1)^2=-26+9+9-1$$

$$3^2(x-1)^2+3^2(y-1)^2-(z-1)^2=-9$$

$$\frac{9(x-1)^2}{9}+\frac{9(y-1)^2}{9}-\frac{(z-1)^2}{9}=-\frac{9}{9}$$

$(x-1)^2+(y-1)^2-\frac{(z-1)^2}{9}=-1$ is hyperboloid of Two sheet with centre $(1,1,1)$

Traces $(x-1)^2+(y-1)^2-\frac{(z-1)^2}{9}=-1$

(i) **xy-plane** $z=0$

$$(x-1)^2+(y-1)^2-\frac{(0-1)^2}{9}=-1$$

$$(x-1)^2+(y-1)^2-\frac{1}{9}=-1$$

$$(x-1)^2+(y-1)^2=-1+\frac{1}{9}$$

$$(x-1)^2+(y-1)^2=\frac{-9+1}{9}=\frac{-8}{9}$$

No solution so, No trace,

(ii) **xz-plane** $y=0$

$$(x-1)^2+(0-1)^2-\frac{(z-1)^2}{9}=-1$$

$$(x-1)^2+1-\frac{(z-1)^2}{9}=-1$$

$$(x-1)^2-\frac{(z-1)^2}{9}=-1-1=-2$$

$\frac{(z-1)^2}{9}-(x-1)^2=2$ is hyperbola

(iii) **yz-plane** $x=0$

$$(0-1)^2+(y-1)^2-\frac{(z-1)^2}{9}=-1$$

$$1+(y-1)^2-\frac{(z-1)^2}{9}=-1$$

$$\cancel{-}\left(\frac{(z-1)^2}{9}-(y-1)^2\right)=-1-1=\cancel{-}2$$

$\frac{(z-1)^2}{9}-(y-1)^2=2$ is hyperbola

lecture 40

Spherical Trigonometry

Main branch of mathematics

(1) Arithmetic (ii) Algebra (iii) Geometry

Geometry

the mathemetical study of properties, measurement, relationship of points, circles, line Planes, surfaces, angles etc.

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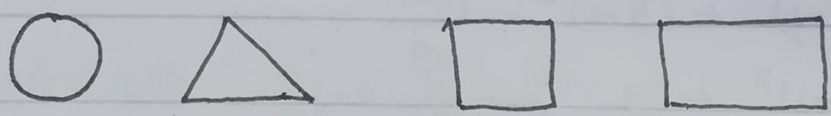
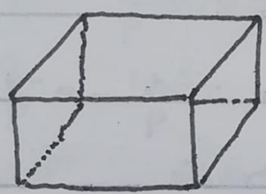




Two-dimensional figures

It is a figure That has length and width but no depth. In ~~x~~-axis and y-axis e.g. circles, squares, Rectangle, Triangles etc

Three-dimensional figures

It is a figures that has length, width and depth. x-axis, y-axis z-axis

e.g. cube, ~~prime~~ prism, sphere, cylinder, cone.

Types of Geometry

(1) Plane geometry (ii) Solid geometry

(iii) Spherical geometry

1 Plane geometry

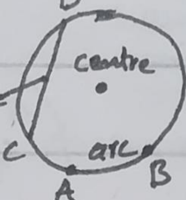
which deals with only two dimensions.

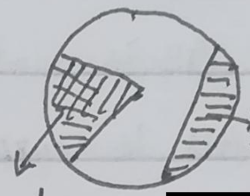
e.g circle

r = radius

d = diameter

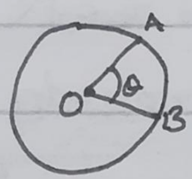
$d = 2r$





$C = 2\pi r = \pi d$

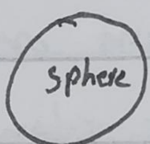
$A = \pi r^2$



l = length of arc $= r\theta$

Area of sector $= \frac{1}{2} r^2 \theta$

(2) Solid Geometry $\longrightarrow$



$A = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

which deals with all Three-dimensions.

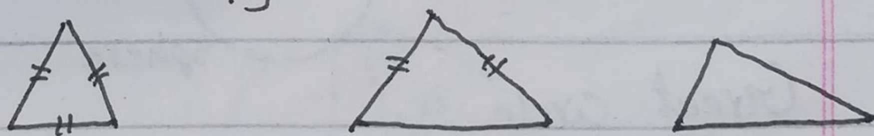
e.g. cube, sphere, cylinder, cone, etc.

Trignometry

It is the branch of geometry that studies relationships involving length and angles of Triangles.

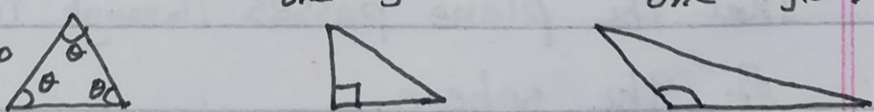
By side

e.g



Equilateral △ Isosceles △ Scalene △

By angles



Acute △ Right △ Obtuse △

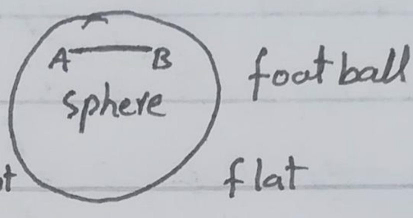
Spherical geometry

The study of figures on The surface of The sphere.

Surface of sphere is not flat

AB is not a straight line

AB is curve because surface of sphere is not flat

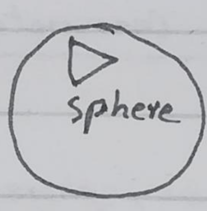


Spherical Trignometry

It deals with triangles drawn on The surface of sphere.

not plane triangle

all sides are arc



Lecture – 41

Intersection of a plane and a sphere

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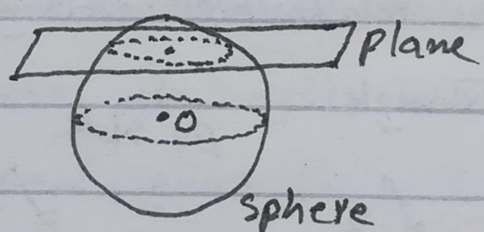
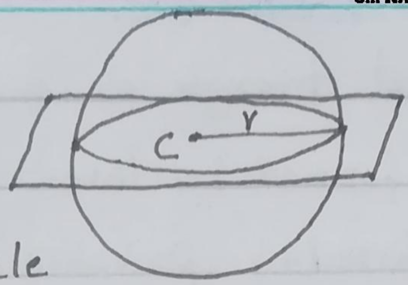


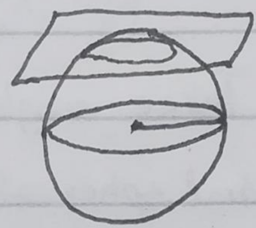
Great circle

A great circle of a sphere is The intersection of the sphere and a plane such that the plane passes Through the centre of the sphere.



Small Circle

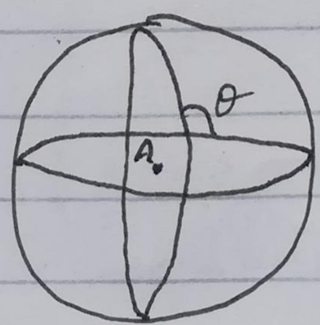
A small circle of a sphere is the intersection of the sphere and a plane such that the plane does not pass through the centre of the sphere.



Shortest distance between two points on the surface of sphere/earth

Spherical angle

It is the angle between two intersecting arcs on a sphere, and is measured by the angle between the planes of the great circles containing the arcs.



Spherical Triangle

A spherical triangle is a part of surface of sphere formed by Three great circular arcs intersecting pairwire in Three vertices.

Spherical Triangle

1 The Three sides are all The arcs of great circles.

(2) Any Two sides are ~~Two~~ together longer Than The Third side

(3) The sum of Three angles is greater Than 180°

(4) Each individual spherical angle is less Than 180°

Plane triangle

(1) The Three sides are straight lines on The surface of plane

(2) Any two sides are together longer Than The Third side

(3) The sum of Three angles is always equal to 180°

(4) Each individual angle is less Than 180°

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lecture 42

The cosine formula

AOB, AOC and BOC are Three planes

l, m, n are unit vectors. so $r = |l| = |m| = |n| = 1$

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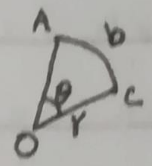
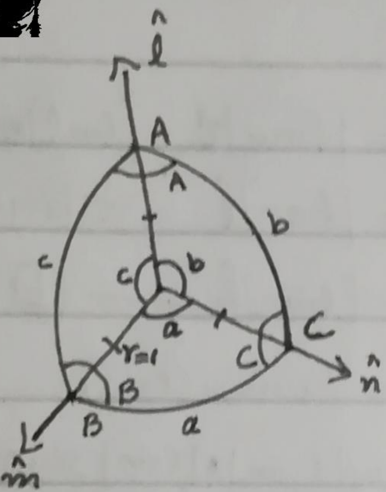




$l = r\theta$

$l = b$
$r = 1$

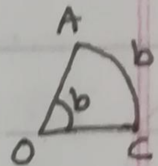
$b = (1)\theta$

$b = \theta$

so,

$a = \theta$

$L = \theta$



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Using dot and cross product

$$(l \times m)\cdot(l \times n) = \begin{vmatrix} l\cdot l & l\cdot m \\ m\cdot l & m\cdot n \end{vmatrix} \text{ ——— } 1$$

Take R.H.S $= \begin{vmatrix} |l||l|\cos 0 & |l||m|\cos b \\ |m||l|\cos c & |m||n|\cos a \end{vmatrix}$

$$= \begin{vmatrix} 1 & \cos b \\ \cos c & \cos a \end{vmatrix} = \cos a - \cos b \cos c \text{ — } (2)$$

Take L.H.S $= (l \times m)\cdot(l \times n)$

$= |l \times m||l \times n| \cos A$ dote Rule

$= (|l||m| \sin c)(|l||n| \sin b) \cos A$ Cross

$= \sin b \sin c \cos A$ ——— (3)

Value of (2) and (3) put in (1)

$$\sin b \sin c \cos A = \cos a - \cos b \cos c$$

$$\sin b \sin c \cos A + \cos b \cos c = \cos a$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

The sine formula

$$(l \times m) \times (l \times n) = [lmn]l - [lml]n$$

$$= [lmn]\hat{l} \quad \therefore [lml]n = 0 \text{ because same vector } l$$

$$(l \times m) \times (l \times n) = [lmn]\hat{l} \quad \text{—— (1)}$$

Take L.H.S $= (l \times m) \times (l \times n)$

$$= |l \times m| * |l \times n| \sin A \hat{l}$$

$$= (|l||m| \sin c)(|l||m| \sin b) \sin A \hat{l}$$

$$= \sin c \sin b \sin A \hat{l}$$

So,

$$\sin b \sin c \sin A \hat{l} = [lmn]\hat{l}$$

$$\sin b \sin c \sin A = [lmn] \quad \text{coffient} - \hat{l}$$

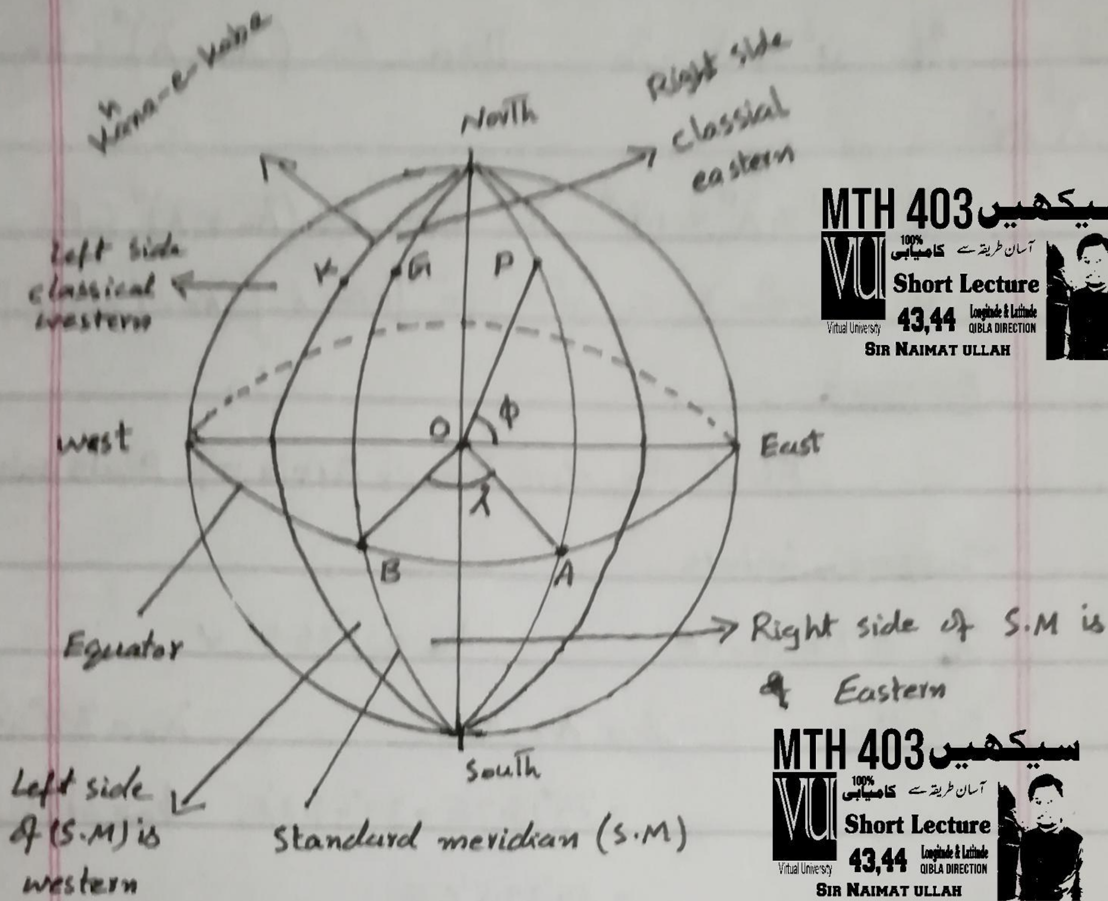
$$\sin a \sin c \sin B = [lmn]$$

$$\sin a \sin b \sin C = [lmn]$$

$$\sin b \sin c \sin A = \sin a \sin c \sin B = \sin a \sin b \sin C$$

Divided by $\sin a \sin b \sin c$

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$



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43,44 Longitude & Latitude QIBLA DIRECTION
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Longitude $= AOB = \lambda$ $\quad P(\lambda, \phi)$

Latitude $= POE = \phi$

Longitude of Khana-e-Kaba $= \lambda_0 = 39^\circ 49' \cdot 2''$ E

Latitude of Khana-e-kaba $= \phi_0 = 21^\circ 25' \cdot 2''$ N

Qibla direction

$\tan i = p - q \quad , \quad i = \tan^{-1}(p - q)$

$p = \dfrac{\sin\phi}{\tan l} \quad , \quad q = \dfrac{\cos\phi \tan\phi_0}{\sin l}$

the classical longitude (l) of a place can be found by

$\lambda^\circ E$ If $\lambda_0 < \lambda^\circ \le 180^\circ$ Then $l = (\lambda - \lambda_0)^\circ CE$

If $0^\circ \le \lambda^\circ < \lambda_0$ Then $l = (\lambda_0 - \lambda)^\circ CW$

$\lambda^\circ W$

If $0^\circ < \lambda^\circ < 180^\circ - \lambda_0$ Then $l = (\lambda_0 + \lambda)^\circ CE$

If $180^\circ - \lambda_0 < \lambda^\circ < 180^\circ$ Then $l = [360 - (\lambda_0 + \lambda)] CE$

<u>Example</u>

Find the direction of Qibla of Badshahi Mosque, Lahore

$\lambda = 74^\circ 18.7' E$ $\quad \phi = 31^\circ 35.4' N$

<u>Solution</u> $l = \lambda - \lambda_0$ $\quad \lambda_0 = 39^\circ 49.2' E$

$= 74^\circ 18.7' E - 39^\circ 49.2' E$ $\quad \phi_0 = 21^\circ 25.2' N$

$= 34^\circ 29.5' CE$

$$P = \frac{\sin\phi}{\tan l} = \frac{\sin(31^\circ 35.4')}{\tan(34^\circ 29.5')} = 0.7624$$

$$q = \frac{\cos\phi \tan\phi_0}{\sin l} = \frac{\cos 31^\circ 35.4' \tan 21^\circ 25.2'}{\sin 34^\circ 29.5'} =$$

$q = 0.5901$

$i = \tan^{-1}(P - q) = \tan^{-1}(0.7624 - 0.5901)$

$i = \tan^{-1}(0.1723)$

$i = 9^\circ 46.7'$ South of west